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EFFECT OF ENTRANCE CONDITIONS ON DIFFUSER FLOW

By J. M. Robertson, M. ASCE, and Donald Ross

HYDRAULICS DIVISION

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Irrigation and Drainage	97, 98, 99, 102, 106, 109, 110, 111, 112, 114, 117, 118, 120, 129, 130, 133, 134, 135, 138, 139, 140, 141 (Discussion: D-XXIII, D-3, D-7, D-11, D-17, D-19, D-25-K, D-29, D-30, D-38, D-40, D-44, D-47, D-57, D-70, D-71, D-76, D-78, D-80, D-86, D-87, D-92, D-96)
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Soil Mechanics and Foundations	43, 44, 48, 94, 102, 103, 106, 108, 109, 115, 130 (Discussion: D-4, D-XXVIII, D-7, D-43, D-44, D-56, D-75 D-86)
Structural	42, 49, 51, 53, 54, 59, 61, 66, 89, 100, 103, 109, 113, 116, 117, 119, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 136, 137 (Discussion: D-51, D-53, D-54, D-59, D-61, D-66, D-72, D-100)
Surveying and Mapping	50, 52, 55, 60, 63, 65, 68, 121, 138 (Discussion: D-60, D-65)
Waterways	41, 44, 45, 50, 56, 57, 70, 71, 96, 107, 112, 113, 115, 120, 123, 130, 135 (Discussion: D-8, D-9, D-19, D-27, D-28, D-56, D-70, D-71, D-78, D-79, D-80)

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PAPERS

EFFECT OF ENTRANCE CONDITIONS ON DIFFUSER FLOW

By J. M. Robertson,[1] M. ASCE, and Donald Ross[2]

Synopsis

An experimental investigation was made of the flow conditions in conical diffusers when preceded by relatively short lengths of straight pipe and a well-faired, large area-ratio contraction. Diffusers having total angles of 5°, 7½°, and 10° were studied when preceded by 2-diameter, 5-diameter, and 9-diameter lengths of straight pipe. The experimental measurements included pressure distributions along the diffusers and velocity traverses at three or more stations. It was found that the area-ratio of the diffuser is the major geometric parameter governing the shape and extent of the boundary layer. The extent of the boundary layer at the entrance to the diffuser, given indirectly by the length-diameter ratio of the preceding straight pipe, is as important a factor in determining the flow as is the diffuser angle. Within the range tested, the pressure efficiency was found to be a function of the product of diffuser angle and effective entrance length; and the energy efficiency, although decreasing with increasing area-ratio, is practically independent of the angle and entrance conditions. It was concluded that the only consideration which should affect the choice of diffuser geometry is the desired factor of safety with regard to separation. As a few of the velocity traverses taken in the experimental program indicated or suggested the occurrence of separation near the end of the diffuser, it was possible to determine the critical values of two velocity-profile form parameters. Methods for estimating the values of these parameters in any diffuser were developed. Thus, this study allows the prediction of the mean flow conditions in conical diffusers, at least within the range of the test variables.

Note.—Written comments are invited for publication; the last discussion should be submitted by January 1, 1953.

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Introduction

A diffuser is a common hydraulic component in which the flow expands and the kinetic energy of a high-velocity stream is converted into potential or pressure energy. A well-designed diffuser can have a fairly high efficiency, whereas one poorly designed will not only have a lower efficiency but also may introduce deleterious flow conditions into the following sections of the system. The type of diffuser flow with which this paper is concerned is that in which the diffuser is preceded by a nozzle or contraction and a relatively short length of straight pipe, as in a venturi meter. Thus, the velocity profile at the start of the diffuser is characterized by a uniform central core and a thin boundary layer at the edges. As the flow proceeds down the diffuser, the boundary layer grows rapidly in thickness until it reaches the center, and the "fully-developed" diffuser flow occurs.

The prediction of the flow conditions in diffusers has long been a major unsolved problem in fluid mechanics. The chief complicating effect is the presence of the large adverse pressure gradient, whose exact influence on the velocity distribution has defied mathematical analysis. Although many investigators have studied the flow of air or water in expanding pipes and channels, some of the basic questions remain unanswered. In most cases, the entrance conditions were either poor, unknown, or otherwise of such a nature as to make the resulting data of doubtful value. Usually, the range of Reynolds numbers was too small to allow scaling of the data, and often certain basic quantities were not even measured. In many cases two-dimensional diffusers, in which the flow is affected by the secondary currents at the corners, were studied.

The present study of diffuser flow was supplemented by a thorough review of the literature (1)[3]. In the well-known, early experimental work of A. H. Gibson (2) (3) the minimum head loss was found to occur for about 5½° (total angle) for circular and square pipes and about 11° for rectangular conduits. The German studies (4) (5) (6) (7) on the flow of air and water in rectangular channels, with diffusion in one plane only, culminated in the work of J. Nikuradse (8). His measurements resulted in the most thorough study of diffuser flow yet reported. Unfortunately, Mr. Nikuradse only presents velocity traverses taken at one station where the flow was "fully developed." He found instability and separation for total angles between 8° and 10°. He presented a semi-rational treatment of the results, which implies that the higher the Reynolds number, the smaller the angle will be at which separation occurs. Experiments similar to the German work were reported by A. M. Vedernikoff (9) who noted instability at the end of his 12° diffuser.

George E. Lyon (10) found the maximum efficiency at 8° for conical draft tubes in which the entering flow was uniform with only a thin boundary layer. In a continuation of Mr. Gibson's work, H. Peters (11) found that, for angles less than 30°, the efficiency is several percent greater with the nearly flat entering velocity profile than with the fully-developed flow. Neither of these studies reveals much about the details of the flow conditions. Other studies

[3] Numerals in parentheses, thus: (1), refer to corresponding items in Appendix I.

of the flow in conical diffusers, with fully-developed entrance flow, are those of G. A. Gourzhienko (12) and A. A. Kalinske, M. ASCE (13).

Other researchers have studied the rate of growth and velocity profile of a boundary layer in an increasing pressure. A. Buri (14) developed a method of analysis based on an approximation to certain terms in the momentum equation of Theodor von Kármán, Hon. M. ASCE. E. Gruschwitz (15) (16) and A. Kehl (17) evolved an empirical method of analysis based upon studies in special channels. The work reported by R. C. Binder (18) in 1947 contains some truly two-dimensional diffuser studies analyzed in a fashion similar to those analyzed by Mr. Buri. A. E. von Doenhoff and N. Tetervin (19) (20) (21) and H. C. Garner (22) have developed semi-empirical methods of analysis which appear to be of limited usefulness (1). A more fundamental approach in terms of the turbulence mechanism has been initiated by K. Fediaevsky (23) (24) and improved by the writers (25).

There appears to be a need for more information on the action of a boundary layer under an adverse pressure gradient. More pressure and velocity measurements, as well as more information on turbulence, are needed. Certainly, the effects of entrance conditions on diffuser flow are not well defined, and the conclusions implied by the Nikuradse analysis need confirmation. This need for more information for use in design analysis was the reason for the tests reported in this paper.

Apparatus and Procedure

Notation.—The letter symbols adopted for use in this paper are defined where they first appear, in the text or by illustration, and are assembled alphabetically for convenience of reference in Appendix II.

Test Facility.—The measurements of the flow in diffusers were obtained in the experimental water tunnel located in the Hydraulics Laboratory of The Pennsylvania State College at State College. This facility, described elsewhere (26), consists of a system in which water flows as great as 10 cu ft per sec can be pumped through aluminum test sections simulating a nozzle, cylindrical pipe section, diffuser, and turn. Fig. 1 shows a typical test setup in which the flow from the 18-in. pipe on the left is accelerated by a nozzle with a discharge diameter of 6 in. A cylindrical pipe section 2 diameters long is separated from the 7.5° diffuser by a gradual transition. The diffuser is followed by a miter turn 12 in. in diameter containing rough turning vanes. The rate of flow through the experimental tunnel was measured by means of a calibrated rectangular weir. The pressure drop across the nozzle was calibrated by comparison with this weir and was then used as a secondary flow standard. The pressure distributions along the walls of the various diffusers were measured by wall piezometer holes connected to differential manometers. Velocity distributions were measured at several cross sections using a cylindrical pitot tube.

Diffuser Test Program.—The nature of the flow in the diffuser is a function of the diffuser angle, the velocity distribution at the entrance, and the Reynolds number of the flow. Diffusers of 5° and $7\frac{1}{2}$° total angle were tested,

preceded by straight pipe-entrance lengths of 2, 5, and 9 diameters, over a pipe Reynolds number—

$$\mathbf{R}_o = \frac{U_o D_o}{\nu} \quad \text{(1)}$$

—range of 0.5×10^6 to 2.5×10^6. (In Eq. 1, U_o is the average velocity in the pipe at the entrance to the diffuser; D_o is the diameter of the pipe at that point; and ν is the kinematic viscosity of the fluid.) A 10° diffuser was tested with 2-diameter and 5-diameter lengths of pipe preceding it. The use of the

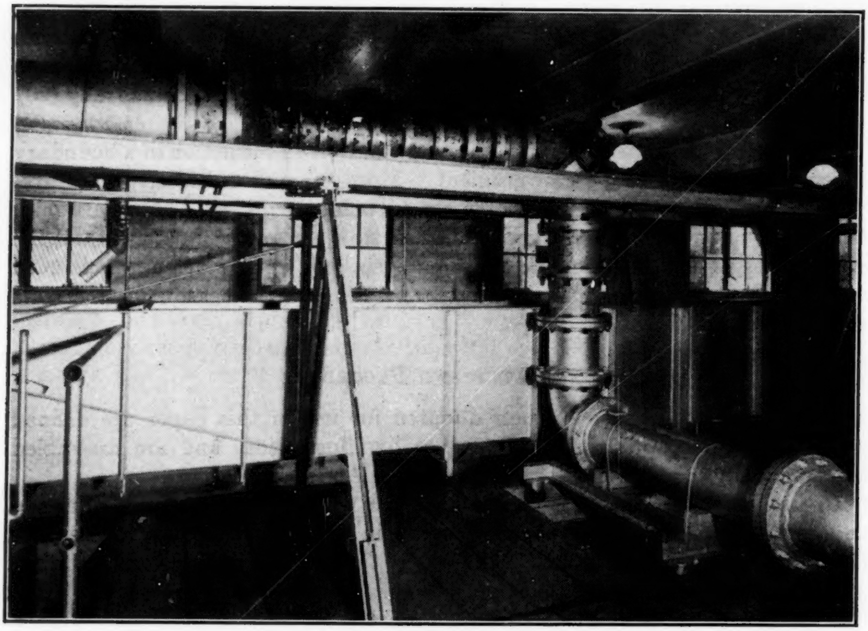

FIG. 1.—EXPERIMENTAL WATER TUNNEL

different entrance pipe lengths gave different boundary-layer thicknesses at the diffuser entrance. The effective entrance length L was greater than the nominal length because the boundary layer had an appreciable thickness at the end of the nozzle (27) and because it continued to grow in the transition section before the effective start of the diffuser. Correcting the nominal lengths for these two factors yielded the values of the effective entrance lengths listed in Table 1.

The pressures were measured throughout the length of the diffuser, over the entire range of Reynolds numbers, and velocity traverses were taken at three or five stations at about three different Reynolds numbers.

Procedure.—The raw experimental data consisted of readings of differential manometers connected between wall piezometers or between the static and dynamic holes of a pitot tube. The wall-pressure distribution data were handled by converting them into pressure coefficients (differences in pressure

between a piezometer and a standard reference piezometer divided by the straight pipe velocity pressure) which were than grouped according to the Reynolds number range in which they were obtained. This procedure yielded average values of the pressure coefficients C_p at various locations in the diffuser for certain Reynolds numbers. Because the data were grouped, these coefficients have an accuracy somewhat greater than that of any individual test reading. The velocity traverse data yielded values for the maximum velocity u_1, the estimated disturbance thickness δ and the displacement thickness δ^*, and the momentum thickness θ, obtained through integration of the velocity profile according to the following relations:

TABLE 1.—EFFECTIVE ENTRANCE LENGTHS (IN DIAMETERS)

Length[a]	5°	7½°	10°
2	3.3	3.5	3.6
5	6.2	6.4	6.4
9	10.1	10.4	....

[a] Nominal length, in diameters.

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_1}\right) dy \qquad (2a)$$

and

$$\theta = \int_0^\delta \left(1 - \frac{u}{u_1}\right) \frac{u}{u_1}\, dy \qquad (2b)$$

Eqs. 2 are more significant measures of the boundary-layer thickness than the apparent or disturbance thickness δ. The displacement thickness (Eq. 2*a*) represents the distance by which the wall streamline is effectively displaced into the fluid in so far as it affects the flow outside the boundary layer, and the momentum thickness (Eq. 2*b*) is a measure of the momentum deficiency of the boundary layer.

TEST RESULTS

The experiments indicated that the action of a diffuser on the nearly uniform entering flow is to increase the rate of growth of the boundary layer and to change the form of the velocity distributions, while lowering the velocity in the uniform central core. This action is illustrated in Fig. 2, which shows velocity and pressure changes for a typical diffuser.

Pressure Variations.—All the diffuser-pressure measurements were made with reference to a piezometer near the downstream end of the working section. These measurements were reduced to pressure coefficients—

$$C_p = \left(\frac{\Delta p}{\frac{1}{2}\rho\, U^2_o}\right) \qquad (3)$$

—and handled in the semi-statistical method previously referred to. For a fundamental presentation, the pressures should be referred to that at the actual or geometric start of the diffuser, defined as the location in the transition section where the projection of the cylindrical entrance pipe wall intersects the projection of the conical diffuser wall. Therefore, the pressure coefficients were corrected for the pressure loss between the reference piezometer and the

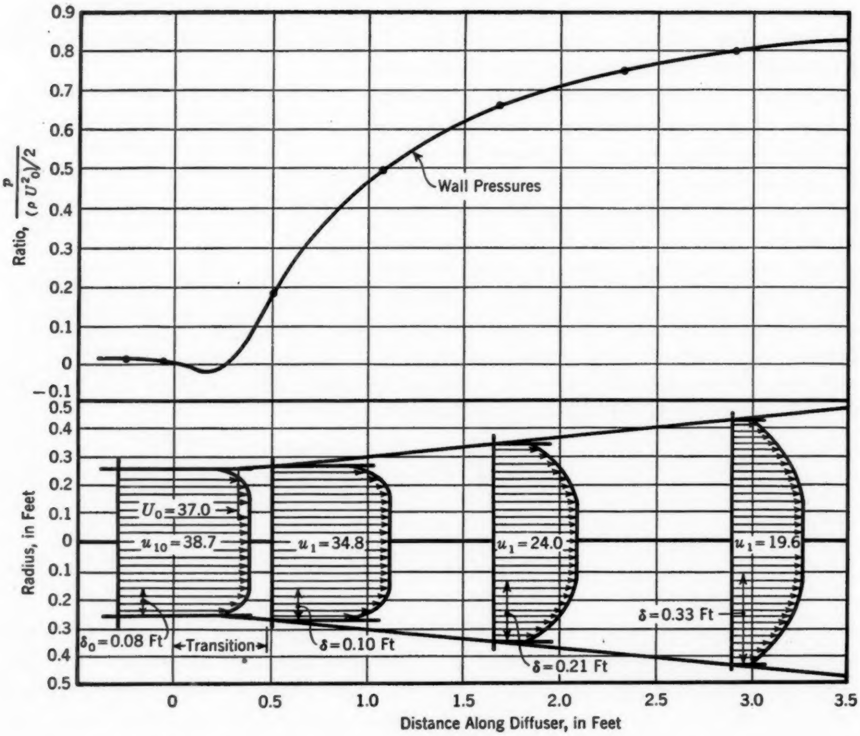

FIG. 2.—FLOW CONDITIONS IN A 7.5° DIFFUSER (FIVE DIAMETERS OF STRAIGHT SECTION PROCEEDING THE TRANSITION; $R_o = 2 \times 10^6$)

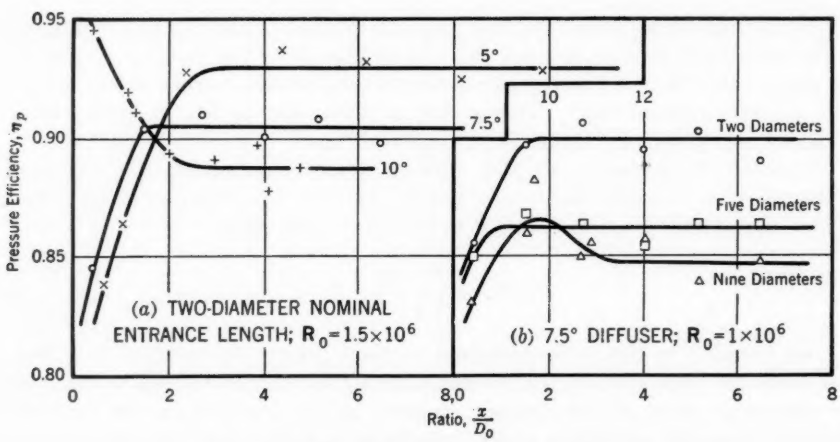

FIG. 3.—PRESSURE EFFICIENCIES ALONG SEVERAL DIFFUSERS

geometric start of the diffuser, as computed from the rate of pressure drop measured in the straight pipe (27). The resultant pressure coefficients are summarized elsewhere (28). Where data were available from more than one test series, the individual results were averaged and weighted in favor of the more reliable data.

The pressure regained in a diffuser is essentially governed by the Bernoulli equation and the equation of continuity. As the diffuser expands, the average velocity must decrease to maintain continuity, and the pressure must rise as kinetic energy is converted into potential energy. If the fluid were ideal, the pressure coefficient could be obtained directly from the geometry of the diffuser:

$$\frac{\Delta p_i}{\frac{1}{2}\,\rho\, U^2_o} = 1 - \left(\frac{D_o}{D}\right)^4 \qquad (4)$$

in which Δp_i is the "ideal" pressure difference between the geometric start of the diffuser and the pressure at the point in question; ρ the fluid density; U_o the average velocity in the entrance pipe; D the section diameter; and D_o the entrance pipe diameter. (Throughout this paper the conduit is treated as being horizontal. For treating inclined conduits p should be replaced by $p + \gamma\, z$.) Through the action of shear forces, some energy is lost and the kinetic energy content of the flow is altered, resulting in a pressure coefficient lower than the ideal value. The ratio of the actual pressure coefficient C_p to the ideal value is the diffuser-pressure efficiency η_p, a useful quantity for diffuser analysis. The manner in which the pressure efficiencies vary with the distance along the diffuser is illustrated in Fig. 3. Although there may be a slight downward trend, the efficiency is essentially constant beyond a distance of 2 or 3 diameters from the start of the diffuser. From these and similar curves, terminal values of diffuser-pressure efficiency were evaluated for each of the eight test setups and for each Reynolds number.

The manner in which diffuser-pressure efficiencies vary with the working-section Reynolds number is shown in Fig. 4. Over the range for which tests were made, the efficiency increases with increase in Reynolds number in the same fashion as the drag of a streamlined body decreases, but in disagreement with the implications of the Nikuradse theory. Fig. 5 is a contour plot showing the terminal pressure efficiency as a function of effective pipe-entrance length and diffuser angle at a Reynolds number of 1.5×10^6. As was to be expected, the efficiency decreases with increase in either of these quantities. That the diffuser angle and pipe-entrance length should be of nearly equal importance in determining the diffuser efficiency was not expected, but that this is approximately the case is shown in Fig. 6. This interpolation curve shows the pressure efficiency plotted against the product of diffuser angle β and the effective pipe-entrance length, L/D_o. All the experimental points lie close to a single curve, and this curve may be used to predict the diffuser efficiency for a diffuser, between the limits: $5° < \beta < 12°$; $2 < \frac{L}{D_o} < 12$; $12 < \beta\,\frac{L}{D_o} < 60$—and at a Reynolds number of 1.5×10^6.

The pressure efficiency data obtained with the experimental water tunnel established the importance of the entrance conditions on a par with the diffuser

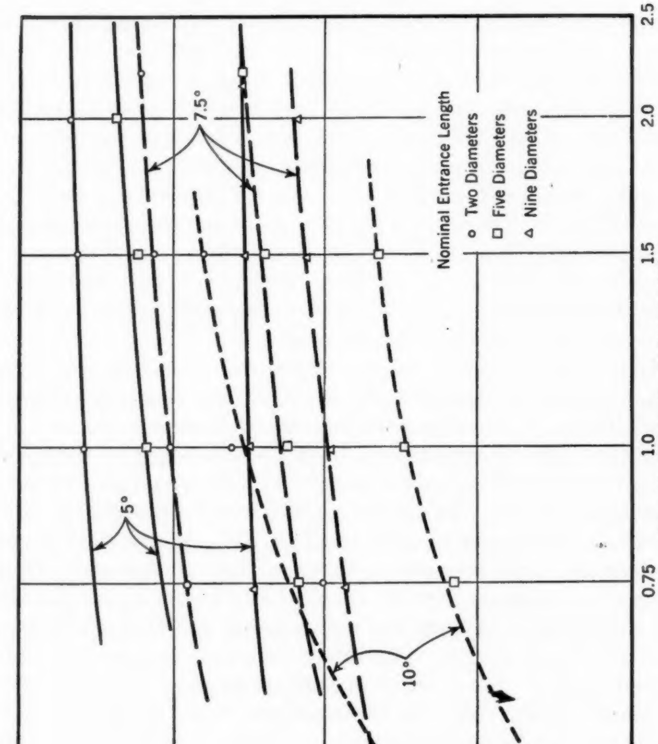

FIG. 4.—TERMINAL PRESSURE EFFICIENCIES AS A FUNCTION OF REYNOLDS NUMBER

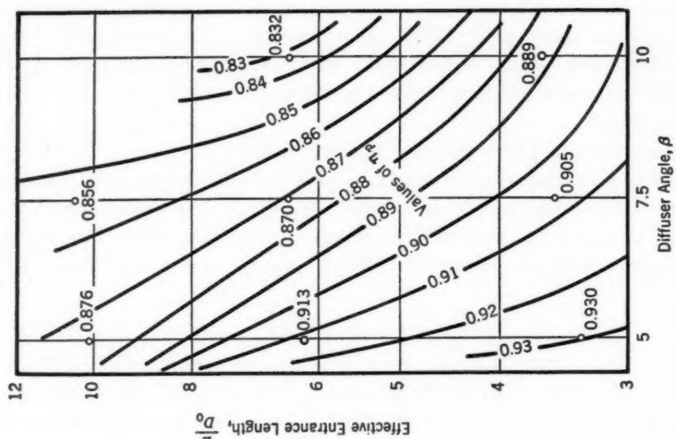

FIG. 5.—PRESSURE EFFICIENCIES AS A FUNCTION OF PIPE-ENTRANCE LENGTH AND DIFFUSER ANGLE ($\mathbf{R}_o = 1.5 \times 10^6$)

angle. Although it had been known that entrance conditions were important, the full extent of this effect had not been realized. For any diffuser satisfying the foregoing limits, the efficiency may be estimated with the aid of Figs. 4 and 6.

None of the diffusers tested exhibited separation, although two of them (10°, 5-diameter and $7\frac{1}{2}$°, 9-diameter) were close to this condition. It is to be expected that the diffuser-pressure efficiency will drop appreciably with the onset of separation and its consequent high energy loss. The proba-

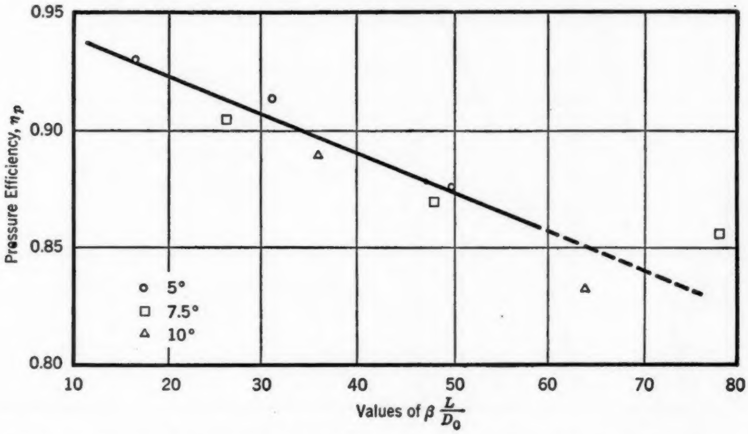

FIG. 6.—PRESSURE EFFICIENCIES AS A FUNCTION OF THE PRODUCT OF THE DIFFUSER ANGLE AND THE EFFECTIVE ENTRANCE LENGTH ($\mathbf{R}_o = 1.5 \times 10^6$)

bility of separation is also a function of the increase in area of the diffuser and consequently no prediction can be based on angle and entrance conditions alone. From these tests, the product of effective pipe-entrance length (in diameters) and diffuser angle should be less than 60 to be reasonably certain of avoiding separation at $\mathbf{R}_o = 1.5 \times 10^6$.

Velocity Distributions.—Although pressure distributions give an indication of the action of a diffuser, it is not possible to obtain a complete picture without velocity-distribution measurements at various planes in the diffuser. The change in shape of the velocity profile has already been depicted in Fig. 2. Fig. 7(*b*) presents sample traverses at various stations in a 5° diffuser preceded by a 5-diameter pipe-entrance length at an entrance Reynolds number of 1.5×10^6. Besides showing the change in shape of the velocity profile as the flow proceeds down the diffuser, these curves show the increase in boundary-layer thickness and decrease in core velocity. These profiles are similar to those reported by other investigators.

The major objective of the diffuser flow studies was to determine the variation of the velocity profiles with the distance from the start of the diffuser, diffuser angle, entrance conditions, and Reynolds number. It was found that, within the accurary of the experiments, the effect of the Reynolds number on the shape of the velocity distributions was negligible. This is illustrated by the data presented in Fig. 7(*a*) for three locations in a 7.5° diffuser. The scatter is typical of the velocity measurements.

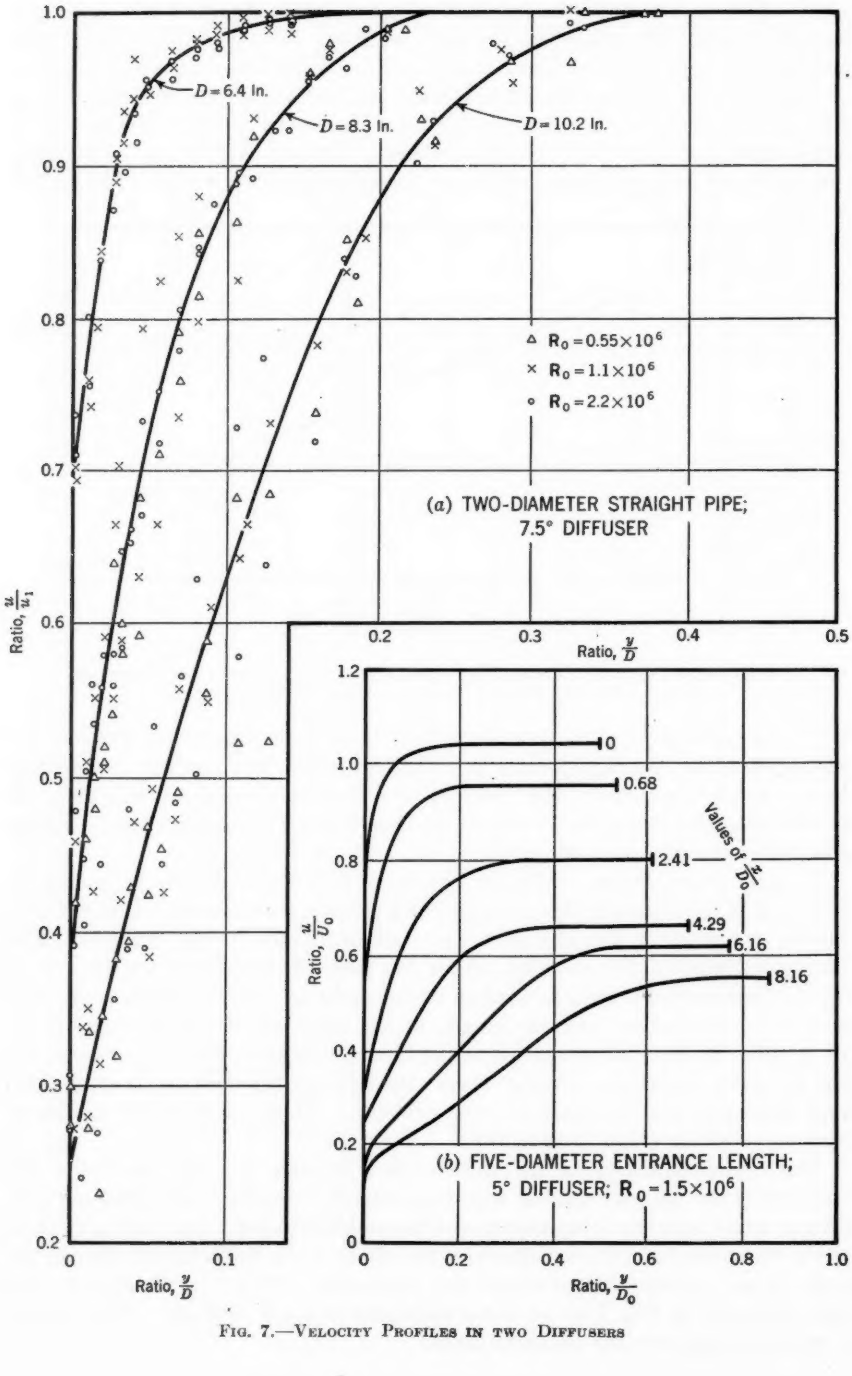

FIG. 7.—VELOCITY PROFILES IN TWO DIFFUSERS

As the function of a diffuser is to expand the flow from one area to another, it is logical to compare velocity traverses in different angle diffusers at the same relative section areas or diameters. Fig. 8(a) presents traverses taken in the three diffusers for the same entrance conditions, at locations having areas 1.87 and 2.85 times the straight pipe area. (This latter ratio is called the area-ratio, A_R.) As was to be expected, the difference between the three angles is marked. The 5° diffuser has the thickest boundary layer, whereas the 10° diffuser has the lowest apparent wall velocity. The angle effect, however, is considerably less than the area-ratio effect.

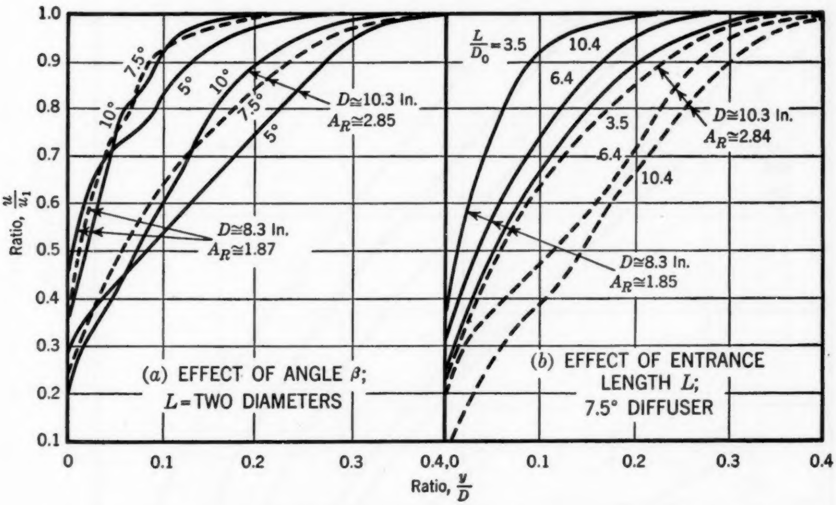

FIG. 8.—EFFECT OF β AND L ON VELOCITY PROFILES AT TWO CROSS SECTIONS ($\mathbf{R}_o = 1.5 \times 10^6$)

The final factor influencing the diffuser velocity profiles is the thickness of the entering boundary layer. This thickness was varied primarily by changing the length of the straight pipe preceding the diffuser. Fig. 8(b) shows traverses at two stations in the 7.5° diffuser for nominal pipe-entrance lengths of 2, 5, and 9 diameters. These curves show a marked increase in effect with increase in entrance boundary-layer thickness. Qualitatively, the pipe-entrance length and diffuser angle appear to be about equally important in determining the velocity profile, as they are in determining the diffuser-pressure efficiency.

The characteristics of diffuser flow have been depicted in Figs. 7 and 8. For a quantitative analysis, each traverse must be characterized by the values of certain parameters. The boundary-layer thickness may be expressed as the disturbance thickness, δ. This is a quantity difficult to define, but for analytical purposes it is more convenient to use the displacement and momentum thicknesses as defined by Eq. 2. Other useful properties are the ratio of the maximum to average velocity u_1/U, the ratio α of the true kinetic energy to that based on the average velocity, and some parameter characteristic of the shape of the velocity distribution. Other researchers (18) have found that the

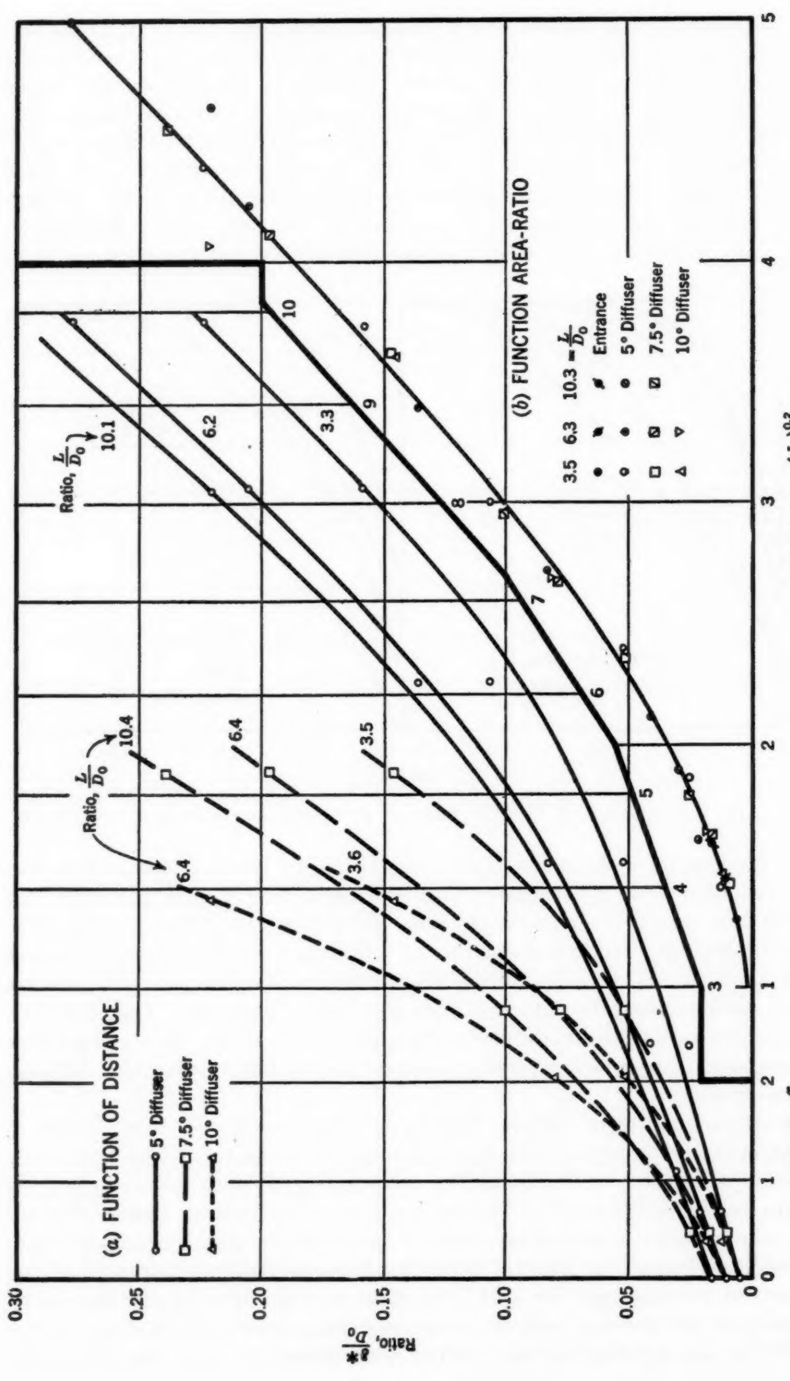

FIG. 9.—DISPLACEMENT THICKNESS AS FUNCTIONS OF DISTANCE AND AREA-RATIO

velocity distributions tend to follow a power law of the form:

$$\frac{u}{u_1} = \left(\frac{y}{\delta}\right)^n \quad \text{(5)}$$

The exponent n thus serves as a convenient form parameter. (Although the power-law velocity distribution did not fit the measured velocity distributions too well, it was used because of simplicity.)

For each of the experimental velocity distributions the displacement and disturbance thicknesses and the ratio of the maximum to average velocity

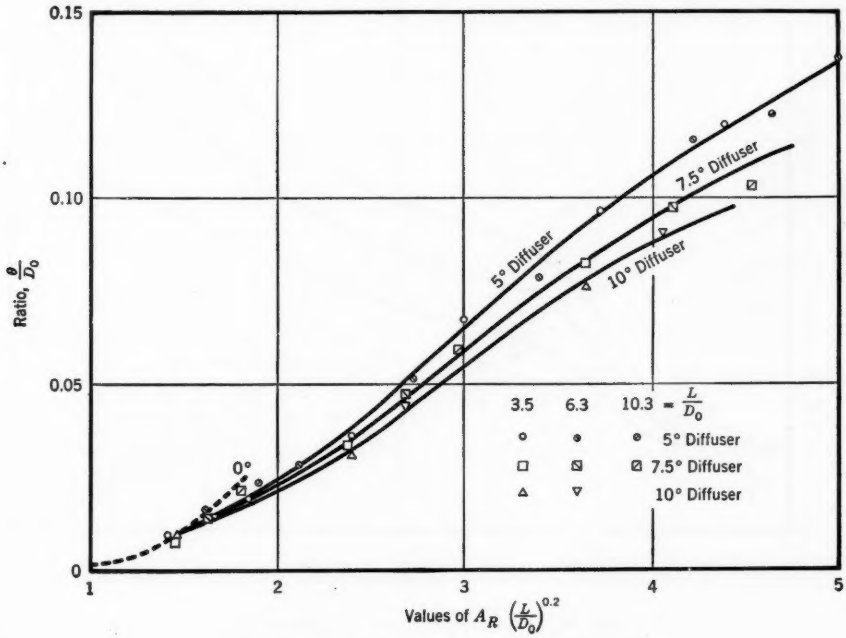

FIG. 10.—GROWTH OF THE MOMENTUM THICKNESS

were obtained by graphical integration. On a log-log plot of the traverses, the best fitting was drawn. The slope of this line gave the exponent n in Eq. 5, and the intercept with $u = u_1$ was chosen to define the disturbance thickness δ. From the exponent n and the ratio δ/D, the kinetic energy coefficient was computed with the aid of the equation:

$$\alpha = \frac{\int^A u^3\,dA}{U^3 A} = \frac{1 - 4\left(\frac{\delta}{D}\right)\left(\frac{3n}{3n+1}\right) + 4\left(\frac{\delta}{D}\right)^2\left(\frac{3n}{3n+2}\right)}{\left[1 - 4\left(\frac{\delta}{D}\right)\left(\frac{n}{n+1}\right) + 4\left(\frac{\delta}{D}\right)^2\left(\frac{n}{n+2}\right)\right]^3} \quad \text{(6)}$$

The measured values of the foregoing quantities are presented elsewhere (28). Inspection of the data indicates that the extent of the boundary layer at any point decreases slightly with increase in the Reynolds number. However, this

effect, which is in agreement with the upward trend in the pressure data, is smaller than the experimental scatter, and, therefore, the data for all Reynolds numbers were averaged.

Boundary-Layer Growth.—A graphical presentation of the variation of the displacement thickness with distance from the start of the diffuser is shown in Fig. 9(*a*), for the various diffusers and entrance conditions. The rate of growth of the boundary layer is much greater in the 10° diffuser than in the 5° diffuser.

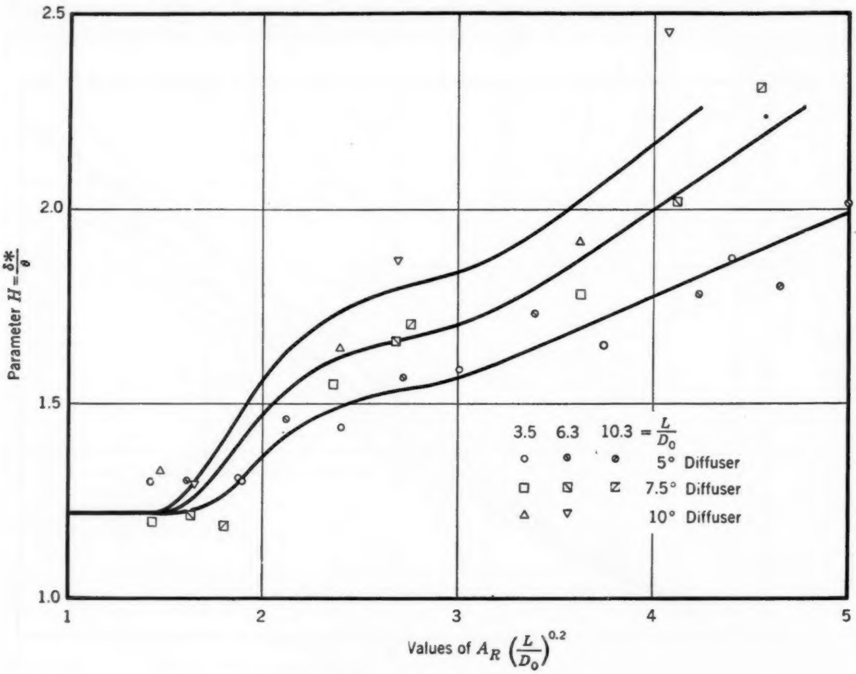

FIG. 11.—VARIATION IN FORM PARAMETER H

As noted previously, one would expect a plot of this thickness as a function of area-ratio to give a more useful comparison of the data for different diffusers. Actually, such a plot eliminates the variation with diffuser angle, leaving only a variation with entrance conditions. Empirically, it was found that the data could be reduced to a single curve by plotting δ^*/D_o against $A_R\ (L/D_o)^{0.2}$ as shown in Fig. 9(*b*). This plot not only summarizes all the diffuser data but also represents the growth of the displacement thickness in the entrance region of straight pipes ($A_R = 1$).

The growth of the momentum thickness is presented in Fig. 10 using the same abscissa. In this case, it is seen that the area-ratio does not account completely for the angle variation, but that the variation with entrance conditions is satisfactorily compensated. The straight pipe growth, shown by the dashed line, is seen to diverge from the other curves in the manner expected. The residual effect of diffuser angle is probably due to the effect of wall shear on the boundary-layer growth.

The ratio H of the displacement thickness δ^* to the momentum thickness θ is a common form parameter used to characterize the velocity profile and to indicate the proximity to separation. This is easily computed from the data presented in Fig. 9(*b*) and Fig. 10, and the resulting values are plotted in Fig. 11. The scatter of these data is so large that it is difficult to define average curves. However, the trend-curves shown in this figure were obtained from the average curves in Fig. 9(*b*) and Fig. 10. These curves are reasonably good for values

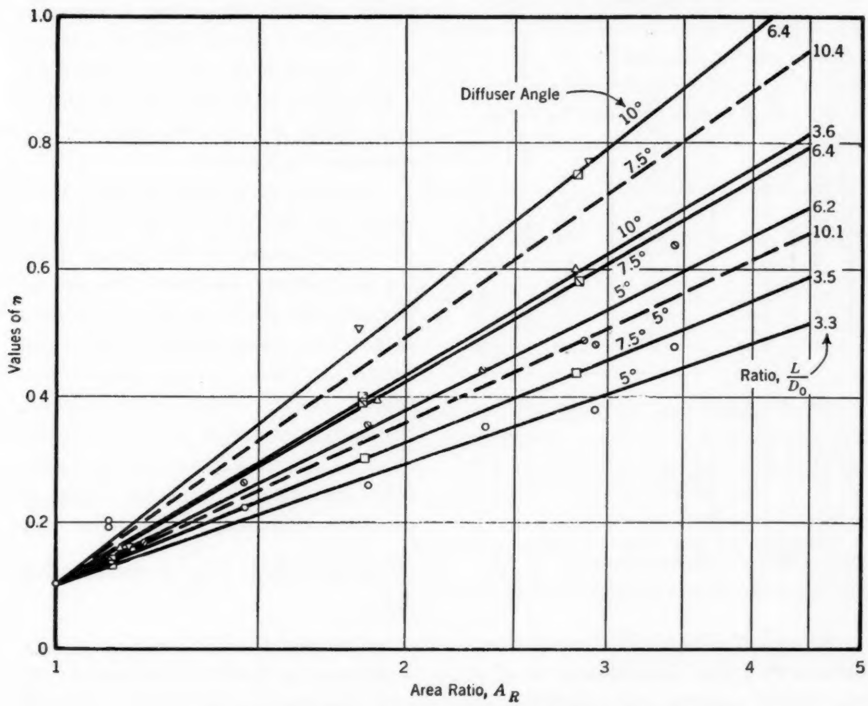

FIG. 12.—VARIATION IN THE POWER-LAW EXPONENT n

of H less than 2. As might be expected, the 10° diffuser has the highest H-value, and hence is nearer separation, at any given station for specific entrance conditions.

The results of the power-law analysis of the diffuser velocity profiles are shown in Fig. 12, in which the exponent n is plotted as a function of the area-ratio. Beginning with the value of n which occurs at the end of the straight pipe, the exponent increases directly with log A_R. This logarithmic variation applies to a developing boundary layer, but probably does not hold when the boundary layer completely fills the diffuser. The variation of n with diffuser angle and entrance conditions is not definite, although the product $\frac{L}{D_o}\beta$ seems to determine the position of the curves up to moderate values $\frac{L}{D_o} = 8$ and $\beta = 8°$.

Occurrence of Separation.—Some of the velocity traverses taken near the downstream end of the 7.5° diffuser preceded by a 9-diameter pipe-entrance section and the 10° diffuser with a 5-diameter entrance section suggested that separation was occurring at or near these stations. This condition was indicated by flow asymmetries or zero velocities at or near the wall. In an attempt to establish the conditions under which separation occurs, a large number of traverses were taken near the end of the 7.5° diffuser. The results of these tests, together with those in the 10° diffuser, are presented in Table 2.

TABLE 2.—VELOCITY TRAVERSES INDICATING INCIPIENT SEPARATION

Run	$\mathbf{R}_o$[a]	Comments	H	n
(a) 7.5°; $\frac{L}{D_o} = 10.4$; $\frac{D}{D_o} = 1.68$; $A_R = 2.83$				
M 93	0.8	Unsym.[b]	2.43	0.81
M 70	0.85	Separated	....	....
D 73	1.2	Normal	2.29	0.66
M 90	1.2	Unsym.[b]	2.09	0.59
M 71	1.5	Separated[c]	2.34	0.76
P 1	1.5	Separated	....	....
P 2	1.5	Normal	2.30	0.77
P 5	1.5	Normal	....	....
D 74	1.95	Normal	2.24	0.71
P 3	2.0	Normal	2.35	0.78
P 6	2.0	Normal	....	....
M 91	2.05	Normal	2.42	0.85
(b) 10°; $\frac{L}{D_o} = 6.4$; $\frac{D}{D_o} = 1.68$; $A_R = 2.83$				
E 39	0.5	Normal	2.34	0.70
E 40	0.5	Separated[d]	2.52	0.76
E 41	0.85	Separated[c]	2.57	0.85

[a] Multiply by 10^6. [b] Slightly unsymmetrical. [c] Just separated. [d] Almost separated.

At these two stations, the occurrence of separation was noted in only a fraction of the tests, with the probability seemingly greater at the lower Reynolds numbers. Values of the form parameters H and n are listed in Table 2 for those cases where they could be evaluated. Separation seems to occur for values of H of about 2.4 and for n about 0.8. As noted in the discussion of pressure efficiency, separation can occur for values of $\frac{L}{D_o}\beta$ greater than 60.

Energy Relations in Central Core.—The existence of a flat central core in diffuser flow has been shown in all plots of diffuser velocity distributions. In this region, outside of the wall boundary layer, the transverse velocity gradient is zero, and there are no shear stresses. It follows that there is no energy loss in this region, and that the sum of the static pressure and core-velocity pressure must be a constant. Table 3 gives the static and velocity-pressure coefficients and their sum for the 7.5° diffuser tests. Within experimental accuracy the specific energy of the core is constant, a fact which is of value in relating the velocity distributions to the pressure coefficients in diffuser flow.

Diffuser Energy Efficiency.—The pressure efficiency η_p used in this paper is the ratio of the diffuser-pressure rise to the ideal value for a frictionless diffuser. This efficiency is a simple measure of the effectiveness of the diffuser, as the ideal pressure coefficient is determined solely by the diffuser area-ratio. However, this does not take into account the changes in the kinetic energy of the flow and hence is not a true efficiency. The efficiency of a diffuser as a circuit element is the ratio of the discharge energy flux to the input energy flux, or the

TABLE 3.—ENERGY RELATION IN THE CORE OF A 7.5° DIFFUSER ($\mathbf{R}_o = 1.2 \times 10^6$)

$\frac{D}{D_o}$	(a) PIPE-ENTRANCE LENGTH, 3.5				(b) PIPE-ENTRANCE LENGTH, 6.4				(c) PIPE-ENTRANCE LENGTH, 10.4			
	Pressure coefficient (1)	$\left(\frac{u_1}{U_o}\right)^2$ (2)	Bernoulli constant (3)	Difference[a] (4)	Pressure coefficient (1)	$\left(\frac{u_1}{U_o}\right)^2$ (2)	Bernoulli constant (3)	Difference[a] (4)	Pressure coefficient (1)	$\left(\frac{u_1}{U_o}\right)^2$ (2)	Bernoulli constant (3)	Difference[a] (4)
1.000	0	1.049	1.049	....	0	1.090	1.090	....	0	1.135	1.135	....
1.061	0.179	0.845	1.024	−0.025	0.179	0.898	1.077	−0.013	0.178	0.952	1.130	−0.005
1.362	0.642	0.392	1.034	−0.015	0.615	0.461	1.076	−0.014	0.605	0.528	1.133	−0.002
1.684	0.792	0.250	1.042	−0.007	0.759	0.326	1.085	−0.005	0.749	0.407	1.156	+0.021

[a] Difference from initial value (Col. 3 for $D/D_o = 1.000$).

entering energy minus the energy losses divided by the entering energy:

$$\eta_e = \frac{\alpha_o \dfrac{\rho\, U^2{}_o}{2} - \text{losses}}{\alpha_o \dfrac{\rho\, U^2{}_o}{2}} \quad \ldots\ldots (7)$$

The pressure p is not included in the expression for the entering energy because, in itself, it does not represent a capacity to do work (29). Therefore, the only entering energy is the kinetic energy. Determining the losses from the Bernoulli theorem, the efficiency is

$$\eta_e = \frac{p - p_o + \alpha \dfrac{\rho\, U^2}{2}}{\alpha_o \dfrac{\rho\, U^2{}_o}{2}} = \frac{C_p}{\alpha_o} + \frac{\alpha}{\alpha_o}\left(\frac{D_o}{D}\right)^4 \quad \ldots\ldots (8a)$$

in which the subscript o refers to the entrance conditions. The energy efficiency is related to the pressure efficiency by the formula,

$$\eta_e = \frac{\eta_p}{\alpha_o} + \frac{\alpha - \eta_p}{\alpha_o}\left(\frac{D_o}{D}\right)^4 \quad \ldots\ldots (8b)$$

From Eq. 8*b* it may be seen that for a very large area-ratio diffuser, the energy efficiency becomes less than the pressure efficiency. For small area-ratios, η_e is greater than η_p and is close to unity.

Values of the energy efficiency have been computed from the diffuser measurements using Eq. 8*b*. The variation in η_e with the Reynolds number was found to be less than the variation in η_p because of the slight decrease in α with an increase in Reynolds number. For an average test Reynolds number the energy efficiencies, at an area-ratio of about 3, are listed in Table 4 together with the corresponding values of the pressure efficiency. The energy efficiency is seen to be a more constant factor than the pressure efficiency. A reasonable value for a diffuser efficiency at an area-ratio of three, without separation, is 0.95.

The energy efficiency is higher than the pressure efficiency. Neither of these terms, as measured, completely accounts for the action of the diffuser as a circuit element. These efficiencies are for stations at the end of a diffuser, but the diffuser has an effect on the flow in the following section. Determination of the true diffuser efficiency must include an analysis of the energy changes introduced into the following sections by the diffuser. Thus, if the diffusers in Table 4 were followed by straight pipe, there would be some additional pressure increase and energy loss resulting from the redistribution in the velocities. The resulting pressure efficiency would be higher, whereas the final energy efficiency would be lower.

TABLE 4.—Diffuser Efficiencies at $\mathbf{R}_o = 1.2 \times 10^6$ ($A_R = 3$)

L/D_o	η_p	η_e
$\beta =$ 5°:		
3.3	0.93	0.965
6.2	0.91	0.955
10.1	0.875	0.915
$\beta =$ 7.5°:		
3.5	0.90	0.95
6.4	0.865	0.94
10.4	0.85	0.95
$\beta =$ 10°:		
3.6	0.88	0.945
6.4	0.835	0.94

A common parameter introduced by Mr. Peters (11), and used by other authors, is the diffuser energy conversion efficiency. This is the ratio of the increase in potential energy to the decrease in kinetic energy. This efficiency was not used in analyzing the present data because it is not suitable as a circuit parameter. For a large area-ratio diffuser, the energy conversion efficiency is only slightly less than the energy efficiency. However, for small area-ratios, this efficiency becomes indeterminate.

Summary

The analysis of the diffuser test data obtained in the experimental water tunnel has revealed eight general properties for the flow in the entrance region of a conical diffuser:

1. The effect of the Reynolds number on the flow is small, there being a slight increase in the pressure efficiency and a slight decrease in the extent of the boundary layer with increase in the entrance pipe Reynolds number. There is even less variation in the energy efficiency.

2. The area-ratio of the diffuser is the major geometric parameter governing the shape and extent of the boundary layer.

3. The extent of the boundary layer at the entrance to the diffuser, given indirectly by L/D_o, is as important a factor in determining the diffuser flow as is the diffuser angle β. Within certain limits, the pressure efficiency and the velocity profile are functions of the product of L/D_o times β.

4. The Bernoulli energy (total head) is constant in the uniform central core.

5. Although it decreases with an increase in area-ratio, the energy efficiency is practically independent of angle and entrance conditions, having a value of 0.95 at an area-ratio of three.

6. The boundary-layer displacement thickness is a unique function of $A_R\ (L/D_o)^{0.2}$, the relation also being applicable in a straight pipe. The momentum thickness is a function of the same parameter and of the angle β.

7. Separation occurs for values of H greater than 2.4 and n greater than 0.8. Neither of these form parameters can be correlated too well with geometric parameters near separation. If the product $\frac{L}{D_o} \beta$ is less than 60, separation should not occur for area-ratios up to about 4.

8. The diffuser efficiency is practically constant as long as separation does not occur; therefore, the only factor affecting the choice of diffuser geometry is the desired factor of safety with regard to separation.

The results presented in this paper should enable the design of diffusers for the conditions encountered in water and air tunnel practice and in other large structures in which the type of entrance conditions studied occur.

The manner in which this information would be applied to the design of a diffuser is illustrated in Appendix III where the design analysis is given of the diffuser for the large high-speed water tunnel (30) built at The Pennsylvania State College for underwater ordnance research.

ACKNOWLEDGMENT

The tests and analyses herein described were conducted as part of the design research for the water tunnel built at The Pennsylvania State College by the United States Navy for the use of the Ordnance Research Laboratory (Contract No. NOrd 7958). The project received the active support of H. P. Hammond, M. ASCE, dean of the School of Engineering; Eric A. Walker, director of the Ordnance Research Laboratory; and R. B. Power, project engineer in charge of the engineering and construction of the 48-in. water tunnel. The writers wish to acknowledge the assistance of the personnel of the Ordnance Research Laboratory and of the Hydraulics Laboratory of the Civil Engineering Department, and specifically: A. J. Turchetti, Alfred M. Feiler, A. M. ASCE, P. M. Kendig, W. M. Wachter, A. M. ASCE., Mrs. Irene Sebring, Mrs. Rosemary Schaver, H. W. Bennett, F. E. Shuster, and D. H. Ruhl.

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APPENDIX II. LIST OF SYMBOLS

The following letter symbols, adopted for use in this paper and its discussion, conform essentially with American Standard Letter Symbols for Hydraulics (ASA–Z10.2–1942), prepared by a committee of the American Standards Association, with ASCE participation and approved by the Association in 1942:

A = area;
A_R = area-ratio;
C = coefficient;
C_p = a pressure coefficient defined by Eq. 3;
D = diameter; section diameter;
D_o = diameter of the pipe at the entrance to the diffuser;
H = boundary-layer form parameter; ratio of displacement thickness to momentum thickness;
L = length; effective length of pipe entrance;
n = exponent of velocity distribution power law;

p = pressure per unit area; static pressure intensity (for nonhorizontal conduits p should be replaced by $p + \gamma h$):

- Δp = pressure difference measured from the start of the diffuser;
- Δp_i = ideal pressure difference between the geometric start of the diffuser and the pressure at a given point in question;

R = Reynolds number;
R_o = pipe Reynolds number at entrance of the diffuser (Eq. 1);
U = average velocity, ratio of discharge to area;
U_o = average value of U in the pipe at the entrance to the diffuser;
u = temporal mean velocity at a point in the direction of the tunnel axis;
u_1 = velocity u outside the boundary layer;
x = distance in the direction of flow;
y = transverse distance measured from the surface;
z = elevation above arbitrary datum;
α = kinetic energy coefficient; ratio of the true kinetic energy to that based on the average velocity;
α_o = kinetic energy coefficient at the entrance to the diffuser;
β = total angle of the diffuser;
γ = specific weight of fluid;
δ = boundary-layer disturbance thickness;
δ^* = boundary-layer displacement thickness, Eq. 2a;
η = efficiency:

- η_e = energy efficiency;
- η_p = pressure efficiency; ratio of the diffuser-pressure rise to the ideal value for a frictionless diffuser;

θ = boundary-layer momentum thickness, Eq. 2*b*;
ν = kinematic viscosity of the fluid; and
ρ = fluid density.

APPENDIX III. WATER TUNNEL DIFFUSER DESIGN

The manner in which the information obtained in this study would be applied to the design of a large structure is illustrated in the design of the diffuser of the 48-in. high-speed water tunnel for which these tests were run. The water tunnel is shown in Fig. 13, from which it can be seen that the structure is of considerable size, being nearly 100 ft long. The principal diffuser in the circuit is that which extends from the 4-ft-diameter working or test section (in middle of upper leg) to the turn immediately preceding the propeller pump (in the right-hand part of the lower leg) (30). It is divided into two parts by the first miter turn. The analysis of the experimental studies of diffuser flow presented in this paper indicates three methods of determining the diffuser angle from knowledge of the area-ratio and the entrance conditions. In each case, the analysis is based on the avoidance of separation in the diffuser.

The area-ratio of this diffuser was fixed at $A_R = 4.2$ by the working section diameter of 4 ft and the diameter at the second turn of 8 ft 2 in. The equivalent

or pipe-entrance length was calculated by adding to the 3.5 diameters (14 ft) of the actual working section 2 diameters for the equivalent lengths added by the nozzle and transition sections. Adding an additional diameter, to allow for possible effects of models, gave a total effective working section length of 6.5 diameters.

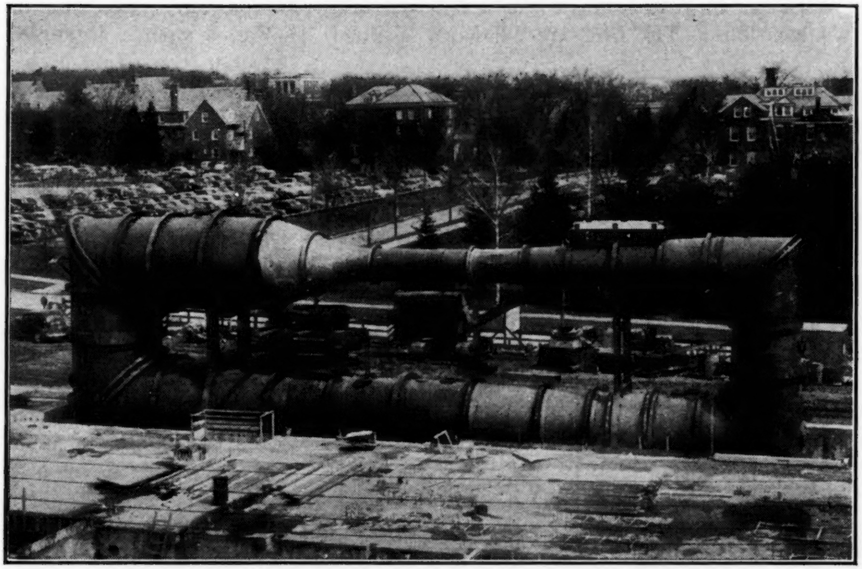

Fig. 13.—Construction View of the Garfield Thomas Water Tunnel

The three methods of diffuser design were based on the exponent n, the form parameter H, and the analysis of the pressure efficiency. The critical value of the power-law exponent n for the occurrence of separation was found to be approximately 0.8. Taking 0.75 as the maximum design value of n, Fig. 12 shows that, at an area-ratio of $A_R = 4.2$ and a ratio L/D_o close to 6.5, the angle should be slightly less than 7.5°. The value of H for separation was found to be 2.4, and 2.3 was taken as a design value. Extrapolating the curves in Fig. 12, and interpolating between 5° and 7.5° angles for $A_R\ (L/D_o)^{0.2} = 6.1$, yields an angle of about 6°. The third way of obtaining the design angle makes use of the fact that separation occurs for values of $\frac{L}{D_o}\beta$ greater than 60, as noted in the discussion of pressure efficiency. Taking the value of 50 as a safe value, the diffuser angle for L/D_o of 6.5 was 7.7°.

The three methods all yield values of the diffuser angle approximating 7°. Each of these analyses included a small margin of safety to account for various possible disturbing factors, such as the effect of the first turn between the two parts of this diffuser. As the H-analysis was inaccurate, less weight was placed on it, and an angle of 7° was chosen for the water tunnel design.

Besides yielding a design for the water tunnel diffuser, the analysis enables a prediction of the flow conditions in the diffuser. The two points of interest are the stations immediately preceding the first and second turns. For these stations the boundary-layer thicknesses and velocity-form parameters, as well as the pressure and energy efficiencies, were computed. The displacement and momentum thicknesses are obtained from Fig. 9(*b*) and Fig. 10. The power-law exponent n can be estimated from Fig. 12 to be 0.60 and 0.68 at the two locations. The pressure efficiency is given by Fig. 6 with a Reynolds number correction based on Fig. 4. The kinetic energy coefficient α is computed from Eq. 6, yielding values of 1.65 and 1.85. The energy efficiency from Eq. 8*b* is 0.95 with a pressure efficiency of 0.905 at a Reynolds number of 12×10^6 (U_o = 30 ft per sec).

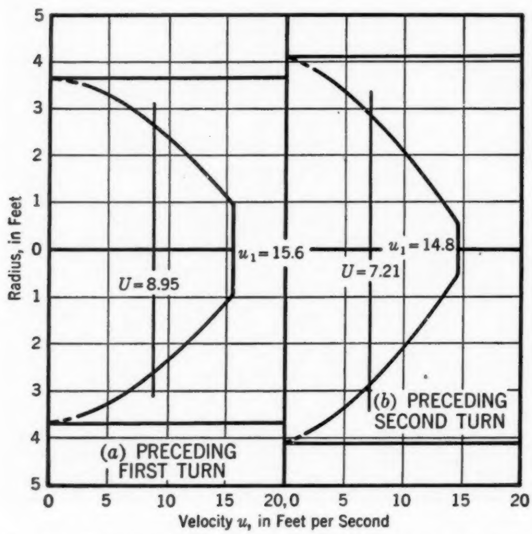

FIG. 14.—PREDICTED WATER TUNNEL DIFFUSER VELOCITY PROFILES, FOR A TEST SECTION VELOCITY OF 30 FT PER SEC

The computations show that at both stations the boundary layer is not fully developed, there is little danger of separation (although at the second station the H-prediction does not agree on this), the pressure and energy efficiencies are constant, and only 5% of the entering kinetic energy is lost. Although the power law is known to be inexact, it was used to predict the approximate velocity distributions at the two stations. These are shown in Fig. 14. Obviously it is expected that the sharp breaks at the outer edge of the boundary layer will be rounded.

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131	June, 1952	"The Allegheny Conference—Planning in Action," by Park H. Martin	Nov. 1
132	June, 1952	"Specifications for Structures of a Moderate Strength Aluminum Alloy of High Resistance to Corrosion," Progress Report of the Committee of the Structural Division on Design in Lightweight Structural Alloys	Nov. 1
133	July, 1952	"Uplift in Masonry Dams," Final Report of the Subcommittee on Uplift in Masonry Dams of the Committee on Masonry Dams of the Power Division, 1951	
134	July, 1952	"Solution of an Hydraulic Problem by Analog Computer," by R. E. Glover, D. J. Herbert, and C. R. Daum	Dec. 1
135	July, 1952	"Application of Electronic Flow Routing Analog," by Max A. Kohler	Dec. 1
136	July, 1952	"Steady-State Forced Vibration of Continuous Frames," by C. T. G. Looney	Dec. 1
137	Aug., 1952	"Construction of the Delaware Memorial Bridge," by Homer R. Seely	Jan. 1
138	Aug., 1952	"The Value and Administration of a Zoning Plan," by Huber Earl Smutz	Jan. 1
139	Aug., 1952	"Nonlinear Electrical Analogy for Pipe Networks," by Malcolm S. McIlroy	Jan. 1
140	Aug., 1952	"Irrigation Water Rights in the Humid Areas," by Howard T. Critchlow	Jan. 1
141	Aug., 1952	"Effect of Entrance Conditions on Diffuser Flow," by J. M. Robertson and Donald Ross	Jan. 1